# 3. Force and gravity

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## Gravity surprises

If the Sun suddenly turned into a black hole, but its mass didn't change (i.e. it stayed the identical mass (about 300,000 times the mass of the Earth) the orbit of the Earth …

*wouldn't change.*

There is a contest underway to see whether a private individual or company can send a rocket to an altitude of 100 km. The first to do this will win $10 million. The energy the rocket uses to get something to an altitude of 100 km, compared to the energy needed to put that same thing into Earth orbit, is …

*thirty times less.*

At an altitude of 100 km, you are virtually at the edge of space, since more than 99.999% of the Earth's atmosphere is below you. At this altitude, the force of gravity is less than at the Earth's surface. It is smaller by …

*about 3%.*

When orbiting the Earth, the head of an astronaut is "weightless." When he sneezes, it will snap back …

*exactly as much as it does when he is sitting on the Earth's surface*

If there were no surprises for you in those sentences, maybe you already know what is in this chapter. For most people, gravity, like many subjects in physics, is full of surprises. If you want to think clearly about space, satellites, and their applications, then it helps to have a deeper understanding.

## The Force of Gravity

A force is a push, something that can compress a spring or get a car moving faster. Forces have magnitude and they have direction. An example is the force of gravity, the force that gives something weight. In fact, we use the same units to describe force and weight: both can be measured in pounds or kilogram.[1] A pound (traditionally abbrevieated “lb”) weighs 0.454 kilogram, i.e. about a half of a kilogram. If you weigh 150 lb, then you weigh 68 kg. Outside the US, people use kilograms for everyday measure of weight.

Gravity wasn't always recognized to be a force. It was once thought to be the natural tendency of all objects to move downward. Isaac Newton was the first person to recognize that weight is simply the force of attraction between an object and the Earth. The famous (and possibly apocryphal) story is that Newton figured this all out when an apple from an overhanging branch fell on his head.[2]

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[1] Kilograms were not originally meant to be units of force. But the public paid no heed. Kilograms are commonly used now to mean weight, not just by the public, but also by physicists. Rather than insist that future presidents adopt the strict convention, this book will simply recognize that they will need to be familiar with the way the term is used out in most of the real world. The kilogram force is simply defined to be the weight of a kilogram of mass, approximately equal to 9.8 Newtons.

[2] William Stukeley, in his “Memoirs of Sir Isaac Newton's Life” pp19-20, said that after dining with Newton on 15 April 1726, that: “The weather being warm, we went into the garden and drank tea, under shade of some apple-trees, only he and myself. Amidst other discourses, he told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasioned by the fall of an apple, as he sat in contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre.” (as quoted at http://myownrainbow.tripod.com/id199.htm)

Voltaire was also a contemporary and admirer of Newton. In his “Letters on the English,” he says: “Having by these and several other arguments destroyed the Cartesian vortices, he despaired of every being able to discover whether there is a secret principle in nature which, at the same time, is the cause of the motion of all celestial bodies, and that of gravity on the earth. But being retired in 1666, upon account of the Plague, to a solitude near Cambridge; as he was walking one day in his garden, and saw some fruits fall from a tree, he fell into a profound meditation on that gravity, the cause of which had so long been sought, but in vain, but all the philosophers, whilst the vulgar think there is nothing mysterious in it. He said to himself, that from what height soever in our hemisphere, those bodies might descend, their fall would certainly be in the progression discovered by Galileo; and the spaces they run through would be as the square of the times. Why may not this power which causes heavy bodies to descend, and is the same without any sensible diminution at the remote distance from the center of the earth, or on the summits of the highest mountains, why, said Sir Isaac, may not this power extend as high as the moon? And in case its influence reaches so far, is it not very probably that this power retains it in its orbit, and determines its motion? But in case the moon obeys this principle (whatever it be) may we not conclude very naturally that the rest of the planets are equally subject to it? In case this power exists (which besides is proved) it must increase in an inverse ratio of thes quares of the distances. All, therefore, that remains is, to examine how far a heavy body, which should fall upon the earth from a moderate height, would go; and how far in the same time a body which should fall from

The reason the story has endured is that it actually makes some sense. Newton was trying to understand what keeps the Moon going in approximate circles around the Earth. Most people thought that was a waste of time to search for an “explanation”; the Moon goes in circles because that is its natural tendency. Newton realized that the Moon was kept in orbit by the same thing that makes apples fall: the force of gravity. Without gravity, the Moon would move in a straight line, and get further and further away; it is the gravity that attracts the Moon that turns that straight path into a circle. In making his theory, he had taken two apparently unrelated observations (apples fall, the Moon orbits) and recognized that the same idea (the "force of gravity") could explain them both. It was the first "unification" of two forces of physics.[3]

Newton realized that the Earth and the apple attracted each other, and that’s what gravity really is. Because the Earth is so large, it is the apple that does most of the moving. (The Earth will actually move up towards the apple by a very small amount.) The Earth and the Moon attract each other. If the Moon were stationary, it would fall towards the Earth. But the Moon is moving rapidly and so even though it is attracted to the Earth, the pull only makes it move in circles, just as the force of a string can make a rock tied to the end move in circles.

In Newton's view, the Moon is constantly falling towards the Earth. But because of its high transverse velocity, it keeps on missing, and the result is a circular orbit. Today, orbiting space satellites use the same principle. They are constantly falling, but like anything moving forward, their path is curved. The forward velocity is great enough that the curving path misses the Earth, and becomes a circle around it.

In modern terms, the force of gravity between two objects is given by

$$F = G\, M_1 M_2 / r^2$$

$F$ is the force, $M_1$ and $M_2$ are the masses of the two objects (in kilograms[4]), $r$ is the distance between them, and $G$ is a constant that makes the units turn out right. If you

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the orbit of the mon, would descend. To find this, nothing is wanted but the meausre of the earth, and the distance of the moon to it.”

Quoted in David Wells, “Curious and Interesting Mathematics”, Penguin 1997, pp 168-168.

[3] In the 1800s, Maxwell unified the electric and magnetic forces, and created a unified theory of electromagnetism. In the 1970s (check), Salam, Weinberg, and Glashow managed to unify electromagnetism and the “weak force” of radioactive decay; the resulting theory is called the electroweak theory. "Grand unified theories" have as their goal to unify the electroweak forces with those of the nucleus and gravity.

[4] An object that has a weight of one kilogram has a mass of one kilogram.

measure the mass in kilograms, the distance $r$ in meters, then to get the force to come out in Newtons, the constant $G$ = 6.7E-11. If you want the force expressed in kilograms, then $G$ = 6.8E-12. (Don't memorize these numbers. Presidents can look them up, or ask nearby physicists.)

There is a strange aspect to the formula: when $r = 0$, the force becomes infinite (since you are dividing by zero). But this is a misinterpretation. The formula is good only outside a sphere of mass $M$, and the $r$ in the equation is the distance to the center of the sphere. So, for example, on the surface of the Earth the correct value for $r$ is the radius of the Earth, $R_E$. And the force of gravity on a mass $m$ is $F = GM_E m/R_E^2$.

**Test of your understanding**: According to the gravity equation, what is the force on a one kilogram object sitting on the surface of the Earth? Before reading the next paragraph -- guess the answer.

You can calculate answer by putting in $G$ = 6.8E-12, $M_E$ = 6E24 kg, $m$ = 1 kg, and $R_E$ = 6.4E6 meters. If you have guessed or calculated the answer, then feel free to check the footnote.[5]

The gravity equation implies that everything that has any mass attracts everything else. But the constant $G$ is so small, that for small objects the force is tiny. The only reason that we notice gravity is because the mass of the Earth is so large. When astronauts stood on the surface of the Moon, the gravity was weak because the mass of the Moon is much less than that of the Earth. On a tiny asteroid, the force is so weak that an astronaut could launch himself into space by merely jumping.

There are many surprises lurking in the gravity equation. Here is one: suppose you take a rocket straight up to an altitude of 100 km. You are moving away from the Earth. So how much does gravity weaken? If you evaluate the gravity equation for the two distances you'll get the answer. But remember – the right value of $r$ is the distance to the center of the Earth. So when you are on the Earth's surface, $r$ = 6370 km.[6] When you are at an altitude of 100 km, the distance has increase to $r$ = 6470 km. That's only a 1.5% increase. The math works out (try it numerically if you are interested) that the force will be decreased by twice this: about 3%. That's the explanation for one of the "surprises" at the beginning of this chapter. For gravity to weaken significantly, you have to go much higher, to distances of thousands of kilometers.

[5] $F = GM_E m/R_E^2$ = 6.8E-12*6E24*1/(6.4E6)$^2 \approx$ 1. So on the surface of the Earth, one kg weighs one kg. (It weighs less than one kilogram on the Moon.) The only reason the number isn't exactly 1 is that we used approximate values for the constants.

[6] The Earth isn't a perfect sphere, so the radius is not the same everywhere. In this equation I used the average radius, which is 6370 km.

## Acceleration

When you push on an object, the object starts moving (assuming there is no opposing force, like friction of the ground). If you keep on pushing, then it goes faster and faster. Acceleration is defined as the rate at which the speed increases. So, for example, when driving a car you might accelerate to 50 miles per hour in 10 seconds. Your acceleration is then 5 mph per second. In physics it is usually more convenient to measure velocity in meters per second = m/s, and then acceleration in the units of m/s per second.

Newton realized that if the force on an object is constant, then so is the acceleration. In equation form:

$$F_N = m\,a$$

Here F is the force (measured in Newtons), m is the mass (measured in kilograms), and a is the acceleration (measured in m/s per second). If you prefer to measure the force in kilograms, then the equation becomes

$$F_{kg} = F_N/g = m\,a/g$$

where the constant $g = 9.8 \approx 10$.

Think about the meaning of these equations. If the motor of your car pushes it forward with a constant force, then the acceleration a = F g/m will be constant. That means that if you look at your speedometer, the arrow will move up at a constant rate from 10 to 20 to 30 to 40 mph. Constant force means constant acceleration.

It also means that if you are at the top of a building and jump off … well, lets assume you are at the top of a building and you drop something off. The force of gravity is pretty constant (we saw that when we calculated how much it changed when you went up 100 km). So that means that the acceleration of the thing you drop will be constant. Suppose you drop an object that has a mass of m = 1 kg. Then the force $F_{kg}$ on it will be 1 kg. Now look at the last equation. If $F_{kg} = 1$ and $m = 1$, we must have $a/g = 1$, i.e. we have $a = g$. So the acceleration due to gravity will be constant, and moreover, it will be 9.8 m/s per second. Every second that the object falls, its velocity will increase by 9.8 m/s. So, after it falls 1 second, it will be moving at 9.8 m/s. After two seconds, it will be falling at 19.6 m/s. After three seconds, 29.4 m/s. After t seconds, its velocity will be[7]

$$v = g\,t$$

[7] This equation will not work if the force is changing. But even for heights of 100 km, we showed the force changed by only 1.6%, so for anything on Earth, even for Mt. Everest, these equations are pretty accurate.

In the movie King Kong (1933 version), Kong took about 9 seconds to reach the ground when he fell from the top of the Empire State Buidling (height 330 meters). That means his velocity upon impact was $v = g\ t = 9.8 \times 9 = 88$ m/s. To convert to mph, multiply by 2.2, since

$$1 \text{ m/s} = 2.2 \text{ mph}$$

So his velocity was 88 x 2.2 = 194 mph.

The equation that relates the height of the building $h$ to the time $t$ it takes to fall is:

$$h = (1/2)\ g\ t^2$$

Solving for t, we get:

$$t = \mathrm{sqrt}(2\ h/g)$$

From these equations (and the numbers I've given you) you can now figure out the height of the Empire State Building.[8]

For this class, I am not asking that you memorize these equations. Learn them if you wish. It is fun to be able to calculate how long it takes things to fall (e.g. divers off cliffs). The only hard part is making sure the units are all correct.

## Terminal velocity: Dropping food from airplanes

In 2002, during the U.S. war in Afghanistan, food was dropped out of airplanes to feed the Afghan people. The food was dropped from an altitude of 10,000 feet ≈ 3000 meters. (The symbol ≈ means "approximately equal to"). How fast was it going when it hit the ground?

Let's use the equations to figure this out. The time it took to fall was:

$$t = \mathrm{sqrt}(2\ h/g) = \mathrm{sqrt}(2 \times 300/9.8) = 24.7 \text{ seconds}$$

and so its velocity would have been

$$v = g\ t = 9.8 \times 24.7 = 2425 \text{ m/s}$$

$$= 2.4 \text{ km/sec} = 5335 \text{ miles per hour}$$

[8] Note that in 1939, when King Kong fell off it, the Empire State Building was only 6 years old. It did not yet have the 250 foot TV antenna on its top.

The speed of sound is only 330 meters per second, so according to these equations it was going 7.3 times the speed of sound, also called *Mach 7.3*.

Suppose the object slows down by friction with the air, and that all its energy goes into heting the object. In Chapter 2 we show that the temperature it reaches Mach number $M$ will be $M^2$ x 300 K. So from friction with the air, the food should have heated up to $7.3^2$ x 300 K = 16,000 K, more than twice as hot as the surface of the Sun! Clearly that didn't happen to the food dropped on Afghanistan. Our calculation was wrong. But why?

We made the mistake of neglecting the force of air as the packages were dropped. Every time the package hits a molecule of air, it transfers some of its energy. The force F of air depends on the area A of the package (more area means it hits more air molecules) and on the velocity that the package is moving. The equation is:

$$F_N \approx (1/2)\, A\, \rho\, v^2$$

where $F_N$ is the force in Newtons. For the force in kilograms, divide by $g$:

$$F_{kg} \approx (1/2)\, A\, \rho\, v^2 / g$$

The symbol ρ in this equation represents the density of air, which is about 1 kilogram per cubic meter. I used the "approximately equals" sign ≈ since the exact force depends on whether the surface is flat or pointed or rounded.

As the object falls faster and faster, the force of air resistance gets greater and greater. Gravity is pulling down, but the air is pushing up. The force of the air resists the gravity; it opposes it. If the object keeps on accelerating, eventually the force of air will match the weight of the object. When that happens, gravity and air resistance are balanced. The object doesn't stop moving, but it stops accelerating, that is, it no longer gains additional velocity. When this happens we say the falling object has reached *terminal velocity*. We can set $F_{kg} = m$ (where $m$ is the mass of the falling object) and solve for $v$ to get the equation for terminal velocity:

$$v = \mathrm{sqrt}(2mg/A)$$

This is a useful equation. A food packet in Afghanistan weighed about $m = 0.1$ kg, and had an area of about $A = 0.3$ m x 0.3 m = 0.1 square meter; g = 9.8 ≈ 10. Putting in these numbers we get for the food packets:

$$v = \mathrm{sqrt}(2\mathrm{x}0.1\mathrm{x}10/0.1) \approx \mathrm{sqrt}(20) = 4.5 \text{ m/s}$$
$$= 10 \text{ mph}$$

So the food floats down relatively slowly. If it went any faster, the upward force (from the air) would be greater than the weight, and slow the fall.

## Falling people, parachutes, and King Kong again

Let's calculate the terminal velocity for a person. We need to estimate the person's area and his mass. The only area that counts is the area hit by air as he is falling. If he is diving, that would be the area of the top of his head. If he is doing a swan dive, it would be the area of the front of his body, roughly his height times his width. To make the numbers simple, let's assume he is 2 meters tall and 1/2 meter wide, giving an area of 2 x 1/2 = 1 square meter. Take his weight to be M = 160 lb = 70 kg. Then, according to our formula, his terminal velocity will be:

$$
\begin{aligned}
v &= \text{sqrt}(2\text{x}70\text{x}10/1) \\
&= \text{sqrt}(1400) \\
&= 37 \text{ m/sec}
\end{aligned}
$$

We can use the conversion factor that 1 meter per second is 2.24 mph to get

$$v = 82 \text{ mph}$$

That's fast, but people have survived falls from great heights into water. (Try to imagine this next time you are going in a car or train at 82 mph.) If they spread out their arms and legs (like sky divers) that increases their effective area, and they can fall even slower. With a parachute, their mass is about the same, but the area can be 30 times larger. Plug that into the equation and see how much it slows the falling person. (Hint: it will be sqrt(30) ≈ 5.5 times slower. Do you see why?)

What about King Kong? Why didn't he slow down? The reason is interesting. If he were 10 times taller than a person, his weight would be 1000 times more. (His volume is 1000 times greater, since he is not only 10 times taller, but also 10 times wider and 10 times thicker.) But his area would have been only 100 times greater. (The area that the air it hitting, to slow him down, is width times height; it doesn't depend on his thickness.) Plug in those numbers (M = 70,000 kg and A = 100 square meters) and you'll see that King Kong would not have slowed down very much. The movie was accurate! (Maybe they had a Physics 10 student consulting for them.)

A diver will slow down less, since the only area that counts (the part that is hitting the air molecules) is the top of his head.

# Automobile fuel efficiency

A moving car feels the force of air on its front, and that tends to slow the car down. To keep going at the same velocity, the engine must make up the lost energy. We'll show that much of the gasoline used by the car is to overcome the force of this air resistance. That is an important fact to know.

Again, we apply the air resistance equation:

$$F_{KG} \approx (1/2)\ A\ \rho\ v^2\ /g$$

The area *A* that we need to use in this equation is the area of the front of the car, equal to the height of the car times its width. Just to get a rough idea, assume the car is about 2 meters by 2 meters, giving *A* = 4 square meters. For the velocity let's take 67 mph. To use this in the equation we have to convert the velocity to meters per second. (The trickiest part of physics is getting all the units right, so they match the right ones for the equation you are using.) We use:

1 meter per second = 2.24 mph

So 66 mph = 67/2.24 m/s = 30 m/s. The density of air is 1 kg per cubic meter. Plugging in these values, we get that the force on the front of the automobile is:

$$F_{kg} = (1/2)\text{x}4\text{x}1\text{x}900/10 = 180 \text{ kg} \approx 400 \text{ lb.}$$

That is a large force trying to slow you down, greater than the friction on the road. To keep the car moving at the same velocity, the engine must exert an equal and opposite force. That takes a lot of gasoline. We'll show in the next section that much of the gasoline you use is to overcome this air resistance. As a result, car designers have worked hard to "streamline" the shape of automobiles. If, instead of hitting the air with a flat surface (as in the old autos from the 1920s) you tilt the front surface, then the force can be much less, since the molecules can bounce off obliquely instead of hitting the front and bouncing straight back. In such a car, the force can be as low as 100 lb.

To a truck driver, the difference can directly affect his income. Maybe you've noticed the smooth curves that some truck drivers have added to the cabs of their trucks to do the same thing. Reducing the force can save substantial money on gasoline.


**Figure: Aerodynamic design for a truck cab**

The top of the cab has had a contoured shape added to it to make the air bounce off smoothly, at an angle, instead of hitting the flat face of the truck head on. (The added shape is called “fairing.”) This is shape change is sometimes called “aerodynamic smoothing” and it saves gasoline. It makes the effective value of “*A*”, the area in the air resistance equation, smaller.

Note also that at half that speed (i.e. if v = 15 m/s = 33 mph) the force is four times less. So you save even more gasoline by driving slower.

## Force and energy

Energy is discussed in detail in Chapter 1. (If you are reading this chapter before you read chapter 1, don’t worry. Energy is just what you think it is. It can be measured in food Calories, or in the physicist unit called Joules, with 1 Calorie = 4180 Joules.)

The relationship between energy and force is remarkably simple. Energy is force times distance. The equation is:

$$E_J = F_N \; D$$

This means that if you push something with a force $F_N$ for a distance $D$, then the energy it takes is $E_J$.

The reason that I put the little subscripts is to let you know that the equation works only if you use the right units. For that equation, $F_N$ must be in Newtons. If you have the force in kilograms ($F_{KG}$), and you need to plug into the equation, you can convert by using $F_N = g\, F_{KG}$ g, with $g$ = 10. Or, you can use the adjusted force equation:

$$E_J = g\, F_{KG}\, D$$

We’ll now use this equation to calculate that a very large fraction of the gasoline used in an automobile is used to overcome air resistance, at least at high speeds.

### optional: Calculation of the gasoline to overcome air resistance

The emphasis on this course is not in doing calculations. However, I’d like to show you how it they are done. So read this section, but relax, you are not going to be asked to reproduce the calculation. It is important to know the result, however.

If you like math, then you might ask yourself if you already see how to calculate the gasoline needed to overcome air resistance. We already calculated the force, and the last equation tells us the energy. We assume the auto is moving at 67 mph = 30 meters per second[9]. At this speed, a typical auto can go about 15 miles on one gallon of gasoline.

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[9] 1 mile per hour is 0.45 meters per second.

In the previous section, we showed the air resistance (the air pushing on the front of the car, trying to slow it down) was a force of about 100 lb = 45 kg. Fifteen miles is 24000 meters.[10] So the work done is:

$$E_J = F_{KG}\, D/g$$

$$= 45\text{x}24000/10$$

$$= 2.2 \text{ x}10^7 \text{ Joules}$$

Another useful energy unit, discussed in Chapter 1, is the food Calorie (known in chemistry as the kilocalorie) with 1 Cal = 4200 joules. So in terms of Calories,

$$E_{CAL} = 2.2 \text{ x}10^7/4200 = 5100 \text{ Cal}$$

There are about 10 Cal in 1 gram of gasoline (see Table 1.2 in Chapter 1), so to provide 5100 Cal takes 5100/10 = 510 grams = 0.51 kg of gasoline. But automobile engines are only about 30% efficient (that is a useful number to remember) so it takes more: 0.51/0.30 = 1.7 kg of gasoline to provide the extra energy needed to overcome the air resistance. This is 0.58 gallons.[11]

In this example, we assumed that the total gasoline usage to go 15 miles was one gallon. Of this, 0.58 gallons is used to overcome air resistance! This is the important result:

***At 67 mph, a typical car can use over 50% of its fuel just to overcome air resistance!***

If you were to drive at 34 mph (half the original speed) then the force would be reduced by a factor of 4, the energy expended against air resistance would be reduced by a factor of 4. That means that in 15 miles, instead of consuming 0.58 gallons to overcome air resistance, you would use only 0.58/4 = 0.15 gallons for air resistance. You would save 0.58 – 0.15 = 0.43 gallons. By reducing your total gas use by this much, instead of consuming 1 gallon in 15 miles, you would consume 1 gallon – 0.43 gallon = 0.57 gallon in 15 miles. Your mileage would be 15 miles /.57 gallons = 26 mpg! ****

*Going slower saves a lot of gasoline, even though you travel the same distance.*

Based on the approach above, can you calculate the gasoline mileage that you would get if you drove at 100 mph? Hint: separate the gas into two parts: the part to overcome air resistance, and the rest. See how much the air resistance part increases.

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[10] 1 mile is 5280 feet and 1 foot is 0.30 meters.

[11] 1 liter is 0.275 gallons; 1 gallon is 3.6 liter; 1 liter = 1.1 quarts. It is worthwhile remembering that ***1 liter is approximately one quart***.

## Centrifugal Force

When the car that you are in goes suddenly around a corner, you feel thrown towards the outside of the curve. The force that you sense is called "centrifugal force." Strictly speaking, you feel this force only when you are in something (an auto or a merry-go-round or a satellite going around the Earth) that is moving in a curve. The car exerts an equal and opposite force on you (sometimes called the centripetal force) to make you move in this curve. [12]

The centrifugal force is easy to calculate for circular motion. Let your velocity be v, and the radius of the circle be R.

$$F = m\, v^2/R$$

### Weightlessness in space

If you are in the Space Shuttle, you will feel two forces: the force of gravity pulling you down, and the centrifugal force (due to your motion around the earth) pulling you up. But they will exactly cancel each other (that's why the Space Shuttle doesn't get pulled any closer to the Earth), and so you will feel "weightless," even though you are still in the Earth's gravity field.

## Earth satellites

It is the centrifugal force that keeps a satellite from falling.[13] From this fact, we can calculate some very basic things about satellites, such as how fast they must move.

In this section, I am going to derive the equation for Earth satellites. You will not be required to be able to reproduce the derivation; it is here primarily for those who are interested. The results, however, are important.

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[12] Many textbooks state flatly that centrifugal force is really just an "imaginary force," and that it doesn't really exist. This is a useful point of view for students who are trying to learn how to do calculations in the context of a non-accelerated reference frame. But as soon as the student advances to calculations in rotating reference frames (usually in a junior or senior level course), the centrifugal force becomes real, as real as Coriolis force. In advanced courses we use centrifugal potentials without adding the extra word "imaginary." In General Relativity theory, even gravity is geometrical (i.e. it is imaginary, the result of accelerated reference frames); it is in the same class as centrifugal and Coriolis forces.

[13] Optional note for experts: This is a technically correct statement when interpreted in the rotating reference frame of the satellite. An alternative and equally correct view would be to say that in an inertial frame (which will have no centrifugal force), the satellite is accelerated downward, turning the otherwise straight motion into circular.

We procede by setting the centrifugal force equal to the force of gravity. The two equations are:

$$\text{gravity force: } F = GM_E m/R^2$$

$$\text{centrifugal force: } F = mv^2/R$$

Setting them equal gives

$$GM_E m/R^2 = mv^2/R$$

If we solve this equation for v, we get

$$v = \text{sqrt}(GM_E/R)$$

R is the distance from the orbit of the satellite to the center of the Earth. The radius of the Earth is 6370 km. To this we must add the height of the satellite. For a "low earth orbit" that is about 300 km, so the total is R = 6670 km= 6.67E6 meters. Putting in the constants (G = 6.68E-11; $M_E$ = 6E24 kg) the equation gives

$$v = \text{sqrt}(6.68\text{x}10^{-11}\ \text{x}6\text{x}10^{24}/6.67\text{x}10^{6})$$

$$= 7750 \text{ m/sec}$$
$$= 7.7 \text{ km/sec}$$
$$= 4.8 \text{ miles/sec}$$

At this altitude, the orbit has a circumference of $2\pi R$ = 26,200 miles, so the time it takes to go around once is 26,200/4.8 = 5400 seconds = 90 minutes = 1.5 hours. So low earth orbit satellites (called "LEOs") orbit the Earth every 1.5 hours. That's a useful number to know.

Satellites don't work if they are stationary. They can't "hover" above a particular point on the Earth. Low Earth orbit satellites (such as Earth imaging and spy satellites) move past their targets on the ground at nearly 5 miles per second. They can't dwell above a particular spot, or they would fall. It's the centrifugal force that keeps them up.

## Geosynchronous satellites

There is a particularly valuable kind of satellite that does appear to dwell above one point on the Earth, and that is the "geosynchronous" satellite. TV satellite dishes are rigidly pointed at these satellites, and they don't have to be repointed. How can that be? Why doesn't the satellite fall?

The answer is that the geosynchronous satellite is moving, but so slowly that it orbits the Earth only once per day. Since it is placed above the equator, it stays above the same location.

**puzzle**: why must it be above the equator? Could it be placed above New York City?

We can use our equations to calculate the height of such a geosynchronous satellite. The calculation will be optional, but you should know the result: geosynchronous satellites must be at the very high altitude of 22,000 miles. That's over 5 times the radius of the Earth!

### Optional calculation: height of a geosynchronous satellite.

Our equation for the velocity of a satellite was

$$v = \mathrm{sqrt}(GM_E/R)$$

The time it takes to go around once is the distance C divided by the velocity

$$T = C/v$$

The distance C is the circumference of the circle $C = 2 \pi R$. Putting these equations together we get

$$T = C/v = 2\pi R/v$$

$$= 2\pi R/\mathrm{sqrt}(GME/R) = 2\pi R^{3/2}/\mathrm{sqrt}(GM_E)$$

Solving for R we get

$$R = (T/2\pi)^{2/3}(GM_E)^{1/3}$$

$$= 4.2E7 \text{ meters}$$

$$= 42000 \text{ km}$$

$$= 26{,}000 \text{ miles}$$

Thus a geosynchronous satellite goes in a circle of radius 26,000 miles. The surface of the Earth is 4000 miles from the center, so the "height" of the geosynchronous satellite is 22,000 miles. That is much much higher than the 200 mile altitude that is typical of a low Earth orbit. This illustrates the main limitation of geosynchronous satellites: they are very far away. They must be very powerful if they are going to broadcast a signal that we can detect on the Earth. A camera on such a satellite must have a very powerful telescope if it is going to photograph the Earth's surface; we'll show in our chapter on light that the diameter of such a telescope must be comparable to a sports stadium if it is to yield high resolution images on the ground; such a telescope is not practical.

## The moonth

Suppose a satellite was so far away that it took a month to orbit the Earth. How far away would that be? We can use our equation for the distance, and put in the time T = 1 month = 28 days = 2.4E6 seconds. Putting this into our equation for R gives a distance of 3.9E8 meters = 240,000 miles.

There is a satellite orbiting the Earth at this distance. It is called the moon. It takes a month to go around (actually, closer to 28 days, which is why I used that value for T). And that, of course, is the origin of the word "month," named after the moon.

## Escape to space

Again, in this section, I'll do the calculations, but the important things to remember are the qualitative results.

Lifting an object takes energy. If it has a mass m (in kg), and you lift it a height h (in meters), then the energy you must use (in Joules) is the force times the distance, i.e. it is

$$E \approx F\ h$$

$$\approx m\ g\ h$$

where g = 10. I used the symbol "≈" because the equation is only good when h is relatively small compared to the size of the Earth. Suppose you want to lift the object to a height of 100 km = 1E5 meters. Then the energy it takes to lift one kilogram is

$$\begin{aligned} E &= m\ g\ h \\ &= 1 \times 10 \times 1E5 \\ &= 1E6 \text{ joules} \\ &= 240 \text{ Calories} \end{aligned}$$

Compare this to the energy that it takes to put a kilogram object into orbit. We showed the orbit velocity is 7750 m/s. So the kinetic energy for orbit is

$$\begin{aligned} E &= (1/2)\ m\ v^2 \\ &= (1/2) \times 1 \times (7750)^2 \\ &= 3E7 \text{ joules} \end{aligned}$$

This is 30 times greater than it takes to simply lift the object to 100 km. (That facct was one of the surprises in the opening of this chapter.) Getting an object 100 km high does not mean you have developed a means of putting such an object into orbit; that takes 30 times more energy.

If you lift the object very high, then its weight (the force of gravity) starts to decrease, and it gets easier to lift. For example, when you lift the object to a height above the surface equal to $R_E$ (so the object is a total distance of $2R_E$ from the center of the Earth) then the force (i.e. the weight) is only 1/4 of what it was on the surface. So it is easier to lift further. If you keep on going, then the force gets even weaker. The result of this weakening is that if you lift the object all the way to infinity, it does not take infinite energy. In fact, this total energy, called the "escape energy", is given by:

$$\text{escape energy:} \quad E = GMm/R_E$$

Did you notice how similar this equation is to the gravity equation? The only difference is that the gravity equation has an $R^2$ in the denominator.[14]

There is another way to send an object to infinity. Instead of lifting it, you can give it a high velocity, i.e. you can try to throw it to infinity. (We are going to ignore any resistance from air.) The energy of motion (the kinetic energy) is given by

$$\text{kinetic energy:} \quad E = (1/2)\ m\ v^2$$

If you give the object an amount of energy equal to its escape energy, then the object will slow down as it rises, but it will keep moving, and it will go all the way to infinity. To see what velocity this requires, we set the kinetic energy equal to the escape energy:

$$GMm/R_E = (1/2)\ mv^2$$

Solving this equation for v, we get the equation for the escape velocity:

$$\text{escape velocity:} \quad v = \text{sqrt}(2GM/R_E)$$

If you compare this equation to the equation for orbital velocity, you'll see that the only difference is the 2 under the square-root sign. To get to infinity, the required velocity is sqrt(2) larger than the velocity needed to put it into orbit. The kinetic energy required (which depends on the square of the velocity) is exactly twice as much.

It is very interesting that the escape velocity does not depend on the mass. Let's put in the numbers:

$$\begin{aligned} v &= \text{sqrt}(2 \text{ x } 6.68\text{E-}11 \text{ x } 6\text{E}24/6.37\text{E}6) \\ &= 11.2\text{E}3 \text{ m/s} \\ &= 11.2 \text{ km/s} \\ &= 7 \text{ miles per second} \end{aligned}$$

If you can get an object moving this quickly, then (ignoring air resistance, and possible attraction from the sun) it will escape to infinity.

At escape velocity, the energy that a one gram (0.001 kg) object will have is

$$\begin{aligned} E &= (1/2)\ m\ v^2 \\ &= (1/2) \text{ x } 0.001 \text{ x } (11.2\text{E}3)^2 \\ &= 63000 \text{ joules} \\ &= 15 \text{ Calories} \end{aligned}$$

[14] This similarity is not a coincidence. If you know some calculus, then you can understand that the energy is $E = \int F\ dr$. If you put in $F = GMm/r^2$ and integrate from r = $R_E$ to r = infinity, then you the result $E = GMm/R_E$ as stated in the text.

That is 20 times greater than the chemical energy in a similar mass of the explosive TNT. That's why it is so hard to get objects into space. If you use chemical fuel, then the fuel weighs substantially more than the object you are sending up.

## Black holes

Black holes are things that are so heavy that the escape velocity exceeds the speed of light. We'll see in Chapter 11 that nothing can go faster than light. That means that nothing could escape a black hole.

We can get the black hole equation by taking the escape velocity equation and setting the velocity equal to the velocity of light, usually called "c". If R is the radius of the black hole (usually called the "Schwartzchild radius") and M is its mass, then we get[15]

$$c = \text{sqrt}(2GM/R)$$

Solving for R, this gives

The Black Hole Equation:

$$R = 2GM/c^2$$

Any object of mass M that has all that mass in a sphere of radius R or smaller, will be a black hole. If we take M = ME (the mass of the Earth), then we conclude that the Earth would be a black hole if its radius were less than

$$\begin{aligned} R &= 2GM_E/c^2 \\ &= 2 \times 6.68\text{x}10^{-11} \times 6\text{E}24/(3\text{x}10^8)^2 \\ &= 9\text{x}10^{-3}\ \text{m} \\ &= 0.9\ \text{cm} \end{aligned}$$

*Thus, the Earth would be a black hole if all its mass were stuffed into a golf ball.*

For the Sun to turn into a black hole, its mass ($M_S = 2\text{x}10^{30}$ kg) would have to be stuffed in a radius

$$\begin{aligned} R &= 2\ G\ M_S/c^2 \\ &= 2 \times 6.68\text{E-}11 \times 2\text{E}30/(3\text{E}8)^2 \\ &= 3000\ \text{m} \\ &= 3\ \text{km} \\ &\approx 2\ \text{miles} \end{aligned}$$

[15] Strictly speaking, this derivation is not correct since we are assuming that the mass is independent of velocity. We are also assuming ordinary ("Euclidean") geometry. However, when the theory of relativity is used to handle these issues correctly, we wind up with exactly the same equation that we get here.

So if the Sun were squeezed into a radius of 2 miles, it would be a black hole.

Remarkably, we think there are stars in which this happens. In some exploding stars, the core of the star, with a mass comparable to that of the sun, gets squeezed into such a small volume. The object in the sky known as Cygnus X-1 is probably a black hole from such a collapse. If you have spare time, look up Cygnus X-1 on the web and see what you find.

Even more remarkably, the Universe itself may be a black hole. That's because the black hole radius for the Universe is about 15 billion light years, and that is approximately the its size. In other words, the Universe appears to satisfy the black hole equation. We'll discuss this further in Chapter 11. But you probably won't be surprised at one inescapable conclusion that follows: we can never escape from the Universe.

## Gravity for finding oil -- and dinosaur-killing craters

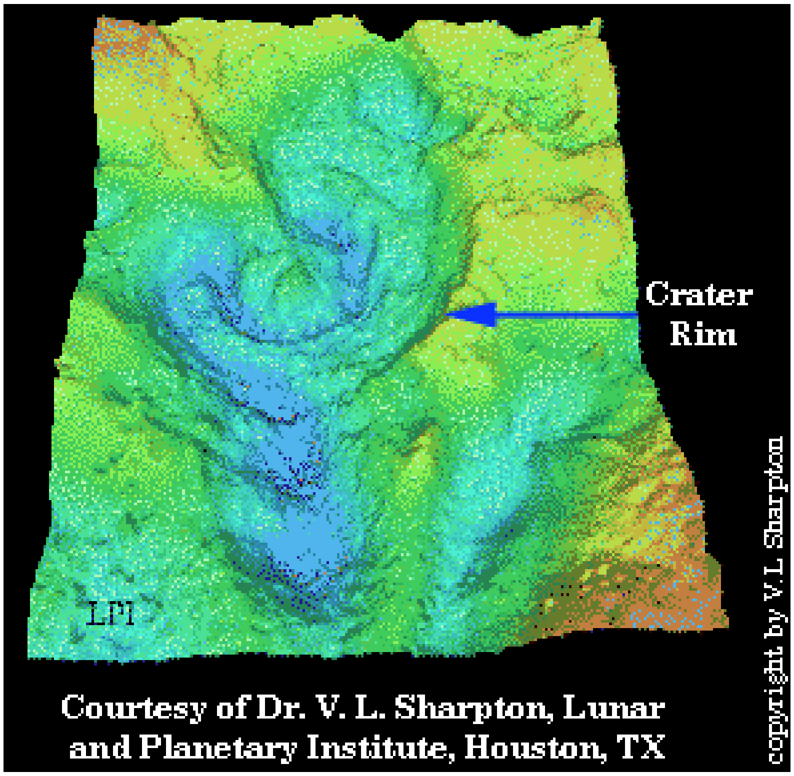
Any two things that have mass will attract each other. Imagine two people, one weighing 160 lb (73 kg) and the other weighing 120 lb, (54 kg) standing 1 meter apart. Their attraction, according to the gravity equation, is

$$\begin{aligned} F &= G\ M_1\ M_2/R^2 \\ &= 6.68\text{x}10^{-11} \text{ x } 73 \text{ x} 54\ /1^2 \\ &= 2.6 \text{ x} 10^{-7} \text{ kg} \\ &= 2.6 \text{ x} 10^{-3} \text{ gram} \\ &= 2.6 \text{ milligrams} \end{aligned}$$

That may not seem like much of a force, but it is about the same as the weight of a mosquito. It can be measured.

Remarkably, measurement of such small forces has important practical applications. If you are standing over an oil field, the gravity you feel will be slightly less than if over solid rock, for the simple reason that oil weighs less, and so its gravity isn't as strong. Such small gravity changes can even be measured from airplanes flying above the ground. An instrument can make a "gravity map" that shows the density of the material under the ground.

Maps of gravity, taken by flying airplanes, are commonly used to search for oil and other natural resources. A more surprising use was to map out the buried crater on the Yucatan peninsula, the crater left behind when an asteroid killed the dinosaurs. The crater was filled in by sedimentary rock that was lighter than the original rock, so even though it is filled, it shows a gravity "anomaly," i.e. a difference from what you would get if the rock were uniform. An airplane flying back and forth over this region made a sensitive map of the strength of gravity, and they produced the map shown below.




The crater shows several concentric circles, with the largest about 200 km in diameter. The inner rings probably formed when the huge crater initially filled in as material from under the crater was forced upward, partially filling it.

## Measuring the mass of the Earth

Have you wondered how we know the mass of the Earth? The first person to determine it was Etvos, in (what year?). He did it in a very indirect way. Even though Newton had discovered the equation of gravity, at the time of Etvos nobody knew the value of the constant G. Etvos determined it by taking several masses in his laboratory and measuring the force of gravitational attraction between them. (It was not an easy experiment.) Once he knew G, he could figure out the mass of the Earth from the known distance to the moon, and the fact that it goes around once every 28 days. In other words, he did the same calculation that we did for the moon, but solved for ME instead of for the distance (which he already knew). When asked what he was doing in his lab, his answer (some people claim) was "weighing the Earth!"

Problem: suppose you learned that the Earth is 150 million kilometers from the Sun. Can use you the principles of this section to calculate the mass of the Sun?

## Gravity on other planets and moons – and in science fiction

Our Moon has about 1/81 of the mass of the Earth. So you might think its gravity would be 81 times less. But its radius is 3.7 times smaller, and that increases the gravity by the square. So the gravity on the surface of the moon is (3.7)2/81 = 1/6 that on the Earth.

What is the surface gravity of an asteroid, which has a radius of only 1 km? We can't say, since we don't know the mass. However, if you assume that the density is the same as for the Earth, then we can derive a simple result: the surface gravity is proportional to the radius.[16] Applied to a 1 km asteroid, this rough rule predicts a surface gravity 1/6378 compared to that on the Earth. So on such an asteroid, a 1 kg mass would have a weight of 1/6378 kg = 0.00015 kg = 0.15 gm = 1/7 of a gram.

One of the most common "errors" in Science Fiction movies is the implicit assumption that all planets in all solar systems have a gravity about equal to that on the Earth. There is no reason why that should be so. Pick a random planet, and you are just as likely to be a factor of 6 lighter (and bouncing around like astronauts on the Moon) or six times heavier and unable to move because of your limited strength. (Imagine a 150 lb person having the effective gravity of a 900 lb body.)

**Numerical exercise**: use the simple radius formula to estimate the surface gravity on the moon. How close is your answer to the correct one that we calculated? Why aren't the answers the same? Can you guess how far wrong the approximate formula must have been when applied to the asteroid?

# Momentum – and Newton's first law

If you shoot a powerful rifle, then the rifle puts a large force on the bullet sending it forward. But the bullet puts a backward force on the rifle, and that is what causes the "kick." (The rifle can suddenly go backwards so rapidly that it can hurt your shoulder. If you don't have your feet firmly planted on the ground, you will be thrown backwards.)

This effect is sometimes given the fancy name "Newton's First Law", and it is stated as "for every action there is an equal and opposite reaction." But we no longer use this old terminology of "action" and "reaction." Instead we say that if you push on an object (such as a bullet), then the bullet also pushes back on you.

The equation for the rifle recoil is simple: the mass of the bullet, times its velocity, is the same as the mass of the gun times its recoil velocity. When the gun is stopped by your

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[16] That's because the mass of the asteroid M = density x volume. Call the density d. The volume of a sphere is $(4/3)\ \pi\ R^3$. So the gravity at the surface is $F = GMm/R^2 = G\ m\ (4/3\ \pi\ R^3\ d)/R^2 = G\ m\ (4/3\ \pi\ d)\ R$. So the weight of a mass of m = 1kg on different planets will depend only on their radius R.

shoulder, then you recoil too – but less, because you have more mass. For the rifle (subscripts R) and the bullet (subscripts B), the equations are:

$$m_B\ v_B = m_R\ v_R$$

The product m v is called the "momentum." One of the most useful laws of physics is called "the conservation of momentum." Before the rifle was fired, the bullet and gun were at rest; they had no momentum. After the rifle fired, the bullet and the rifle were moving in opposite directions, with exactly opposite momenta (the plural of momentum), so the total momentum was still zero.

The same equations work when the comet that killed the dinosaurs crashed into the Earth. To make the calculations easy, assume that before the collision the comet with mass $m_C$ was moving at $v_C$ = 30 km per second (a typical velocity for objects moving around the Sun). Assume the Earth was at rest. After the collision, the total momentum would be the same. The Earth would have mass $m_E$ (that now included the mass of the comet) and have velocity $v_E$. So we can write

$$m_C\ v_C = m_E\ v_E$$

Solving for $v_E$ we get

$$v_E = m_C\ v_C/m_E$$

The mass of the comet is about $10^{19}$ kg.[17] Everything else is known, so we can plug into this equation and get the $v_E$, the velocity of recoil of the Earth:

$$v_E = (10^{19})(30000)/(6\text{x}10^{24})$$
$$= 0.5 \text{ meters per second}$$

That's not much of a recoil, at least when compared to the size of the Earth's orbit (150 million km = 1.5 x$10^{11}$ meters).

Can you estimate how much a truck recoils when it is hit by a mosquito? Assume (from the point of view of the truck, the mosquito (weighing 2.6 milligrams) is moving at 60 mph = 27 meters per second. Assume the truck weighs 5 metric tons = 5000 kg. (Hint: use the same equation, except let the 2.6 milligram mosquito represent the comet.)

Although the the conservation of momentum is one of the most important laws of physics, it is violated in many action movies. For example, if the hero in the Matrix punches the villain, and the villan goes flying across the room, then the hero should go flying backwards (unless he is braced on something big and massive). Likewise, small

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[17] A comet with a radius $R_C$ = 100 km = $10^5$ meters, has a volume of $V_C = 4/3\ \pi\ R_C^3$ (for a sphere) = 4.2x$10^{15}$ cubic meters. Assuming that the comet is made mostly of rock and ice, the density is probably about 2500 kg per cubic meter, so the mass is 2500 x 4.2x$10^{15}$ = $10^{19}$ kg.

bullets, when they hit a person, seem able to impart very large velocities to the person that they hit, so the person goes flying backwards.[18]

## Flying: rockets

Imagine trying to get into space by pointing a gun downward and firing bullets so rapidly that the recoil pushes you upward. Sound ridiculous? That is exactly how rockets work.

Rockets fly by pushing burned fuel downward. If the fuel has mass $m_F$ and is pushed down with a velocity $v_F$, then the rocket (which has mass $m_R$) will gain an extra upward velocity $v_R$ given by the same kind of equation we used for the rifle:

$$v_R = v_F\ m_F/m_R$$

Compare this to the rifle equation, and to the comet/Earth collision equation.

Because the rocket weighs so much more than the expelled fuel (i.e. $m_F/m_R$ is tiny), the amount of velocity gained for every kilogram of fuel used is tiny. The result is that rockets are a very inefficient way to gain velocity. We use them to go into space because in space there is nothing to push against except the expelled fuel.

The equation above gives the velocity *change* when a small amount of fuel is burned and expelled. To get the total velocity given the rocket, you have to add up a large number of such expulsions. Meanwhile, the mass of the rocket (which is carrying the unused fuel) is changing as fuel is used up. If we assume that the rocket started at rest, then the final velocity of the rocket is given by[19]

$$v = 2.3\ v_F \log_{10}(m_R/m_F)$$

This can also be written in terms of the amount of fuel needed:

$$m_F = m_R \text{ x } 10^{(v/2.3vF)}$$

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[18] As a fan of this movie, I explain to myself that according to the script, "reality" is just a computer program called *the matix*. Therefore I can assume that whoever programmed *the matrix* simply forgot to put in conservation of momentum.

[19] If you know some calculus, then you can calculate the final velocity as follows. In the notation of calculus, the change in velocity (which we called $v_R$) is written as dv, and the mass of fuel consumed (which we called $m_F$) is dm, and the mass of the rocket is m. The final velocity $v = \int dv = \int v_F\, dm/m = vF \ln(m_R/m_F) = 2.3\ v_F \log_{10}(m_R/m_F)$. where $m_F$ is the initial mass of the rocket and fuel (mostly fuel) and $m_R$ is the final mass (mostly rocket).

This equation implies that a horrendous amount of fuel is needed to launch a rocket into space. Assume that the space shuttle weighs 20 tons. Assume[20] the fuel velocity is 2 km/sec, and that orbital velocity is 8 km/sec. Then, according to this equation, the mass of fuel needed to reach orbital velocity is

$$m_F = m_R \text{ x } 10^\wedge(8/(2.3*3)$$

$$m_F = m_R \text{ x } 10^\wedge(8/(2.3*2)) = m_R \text{ x}10^\wedge1.7 = m_R \text{ x } 54$$

In other words, the fuel carried must weigh 54 times as much as the payload! For a long time, this problem led people to believe that rockets into space were impossible; after all, how could you even hold the fuel if it weighed 54 times as much as the rocket? The problem was solved by (1) using fuel that had a greater energy per kg than TNT, and (2) using rockets with multiple stages, so that the heavy containers that held the initial fuel never had to be accelerated to the final orbital velocity. For example, for the space shuttle, the final payload (including orbiter weight) is 68,000 kg = 68 tons, and the amount boosters plus fuel weighs[21] 1931 tons, a factor of 28 times larger (but *only* 28 times larger).

Ponder the Space Shuttle a little more. To put 1 gram payload into space requires 28 grams (fuel + container + rocket). Suppose, instead, we built a tower[22] that went all the way up to space. How much energy would it take to haul the ton up to the top, using an elevator? According to the section "Escape to Space", the energy required take a gram of material to infinity is 15 Calories. That's roughly the energy in 1.5 grams of gasoline. So if we had a tower with the elevator, getting to space would take 28/1.5 = 19 times less energy.

Many people have pondered the energy waste from rockets. Although a tower to space seems impossible, it may make sense to hang a cable down from a geosynchronous satellite and use it to haul payload up, an idea once referred to as "project skyhook." The

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[20] If the fuel carries 1 Cal per gram, and all that energy is coverted into kinetic energy, then we can calculate the velocity by assuming it all goes to kinetic energy. 1 Cal = 4200 joules. So 1 kg of fuel would have 4,200,000 joules. If it is all converted to kinetic energy, we would have $(1/2)\ m\ v^2 = 4{,}200{,}000$. Using m = 1 kg, this gives $v^2 = 8{,}400{,}000$. Take the square root to get v = 3000 m/sec = 3 km/sec. Because chemical energy is not efficiently converted to kinetic energy (much remains as heat) the exhaust velocity of 2 km/sec is more typical of what is achieved.

[21] The external tank hold 751 tons of fuel, and there are two solid rocket boosters that weigh 590 tons each, for a total of 1931 tons.

[22] In the Bible, such a tower was attempted in ancient Babylon, and it is also referred to as the "Tower of Babylon." It's goal was to reach heaven. To prevent the Babylonians from succeeding, God made all the works speak different languages. Thus, according to the Bible, is the origin of the multitude of languages spoken by humans. It is also the origin of the verb "to babble."

recent discovery of very strong carbon nanotubes has revived the idea. Arthur C. Clarke used this idea in his science fiction novel "3001: Final Odyssey."

A more reasonable idea is to "fly" to space on an airplane. Airplanes have two attractive features: they use oxygen from the atmospere as part of their fuel (so they don't have to carry it all, as do rockets) and they can push against the air, instead of having to push against their own exhaust. Although it is possible in principle, the technology to achieve 8 to 12 km/sec with airplanes does not yet exist.

## Flying: ion rockets and railguns

A major limitation on the efficiency of rockets, as was described in the previous section, comes from the fact that typical chemical fuels only have enough energy to give their atoms a velocity of one or two kilometers per second. Rockets have been proposed that overcome this limitation by shooting out *ions*, which is a name for atoms that have an electric charge. You can find out a lot about these on the web. Ion rockets are very efficient, but so far nobody has figured out how to make the mass of the expelled ions sufficiently great to be able to launch a rocket from the Earth.

An alternative idea is the "rail gun." A rail gun uses electricity to make a projective move fast. (We'll talk more about this in Chapter 7 Magnetism.) The only fundamental limitation to the speed is the speed of electricity and magnetism, which is near the speed of light. A launch into space using a rail gun would be much like a launch into space from an ordinary gun: all the propulsion would take place in a device that is very big and heavy and remains on the Earth; only the bullet (the rocket) would get into space. A rail gun for launch from the Earth's surface appears to be impractical, partically because of the size required (we can't subject the astronauts to too high an acceleration) and partially because of the problem of atmospheric resistance. Rockets light to start by flying straight up (to get through the atmosphere in its thinnest direction) and then turn and accelerate to high speed only when they are already above most of the atmospheric resistance. Obviously that can't be done with a ground-based rail gun. Rail guns have been proposed for use by the military for ship defense (where the bullet to protect against a cruise missile must move very fast). They may prove to be the most practical means for launching large numbers of vehicles from the surface of the moon, if we ever have a need to do that.

## Flying: airplanes

Airplanes fly by pushing air downwards.[23] Every second, the airplane tends to pick up downward velocity from the Earth's gravity. It stays at the same altitude by pushing

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[23] In most physics books, the lift on the airplane wing is explained by use of a principle called Bernoulli's law. The "derivation" is done using a diagram that typically shows the air trailing the wing as if it is completely undisturbed! The astute student will be bothered by this. How can the air put a force on the wing but the wing not put a force on the air? A careful analysis (done in advanced books in Aerodynamics) shows that to

enough air downward that it overcomes this velocity. The equation is the same as the equation we used for the rifle, and for the rocket:

$$v_{fall} = v_{air}\ m_{air}/m_{airplane}$$

But for the airplane, the $v_R$ needed is the velocity to overcome the pull of gravity. In once second, gravity will give any object a velocity of

$$v = g\ t = 10 \text{ meters per second}$$

So that is the $v_R$ that must be cancelled by accelerating upward, and this is done in an airplane by pushing air downwards. Air is typically a thousand times less dense than the airplane (0.001 gm/cm$^3$), so the $v_{air}$ $m_{air}$ terms must make up for that. To get enough air (i.e. to make mair large) the wings must deflect a large amount of air downward.

The wake of a large airplane consists of this downward flowing air, often in turbulent motion. It can be very dangerous for a second plane if it encougers this wake, since the amount of air flowing downward is large.

## Flying: balloons

The first way that humans "flew" was in hot air balloons, in 1783 above Paris. These make use of the fact that hot air expands. It takes more volume compared to an equal mass of cool air. Another way to say this is that the density of hot air (the mass per volume) is less than that of cool air.

In a liquid or gas, things that are less dense tend to float. That's why wood floats on water (if it has a density less than one gram per cubic centimeter; some woods sink). The heavier fluid "falls" and flows under the less dense object, pushing it upward. Boats float only if their average density (metal hull plus empty space inside) is less than that of water. That's why a boat will sink if it fills with water.

If you fill a balloon with hot air, it will rise until it reaches an altitude at which the density of the surrounding air matches that of the balloon. (Of course, you have to include the mass of the balloon, and any weight that it carries, along with the mass of the air inside.)

Better than hot air are light gases such as hydrogen and helium. Air weighs about 1.25 kg per cubic meter. The same volume of hydrogen weighs about 14 times less, i.e. about

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establish a flow with higher velocity above the wing than below, the far field air velocity distribution is not undisturbed; it fact, it has air deflected downward with a momentum rate equal to the upward force on the wings, as it must to satisfy momentum conservation.

0.089 kg = 89 grams.[24] If we fill a one-cubic-meter balloon with hydrogen, it will tend to float. In fact, the “lift” (upward force) of the balloon is just equal to the difference in weights, so the balloon will have a lift of 1.25 – 0.089 = 1.16 kg. That means that if you hang an object from it that weighs less than one kilogram (including the mass of the balloon skin), it will still go upwards.

Helium gas weighs about 178 gm per cubic meter. So the lift on a helium balloon would be 1.25 – 0.178 = 1.07 kg. Note that even though the helium is twice as dense as hydrogen, the lift is almost as good.

But 1 kg of lift for a cubic meter is not really much. That’s why, despite the cartoons you may have watched on TV, even a large packet of balloons are not enough to lift a 25 kg child.

Hot air balloons have even less lift. If you were to heat the air to 300 C, then it’s temperature in absolute scale would be 600 K. That’s twice its normal temperature, so its density would be reduced by a factor of two. The lift of a cubic meter would be 1.25 – 1.25/2 = 0.62 kg per cubic meter. To lift a person who weighs 100 kg (incuding basket, balloon skin, cables) would take 100/0.62 = 161 cubic meters of hot air. If the balloon were shaped like a cube, the side of such a balloon would be $\sqrt[3]{161}$ = 5.4 meters = 18 feet. That’s why hot air balloons have to be so big.

Salt water is denser than fresh water, so the density difference between it and you is greater; that’s why you float higher in salt water. If water becomes filled with bubbles, its average density can become less than yours, and you will sink. Even strong swimmers take advantage of their buoyancy (the fact that they are less dense, on average, than water). The New York Times on August 14, 2003, had an article describing how a group of boys drowned because they were in bubbly water.[25] Undersea volcanic eruptions have led to bubbly water in the oceans, and in such water even ships will sink.

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[24] Nitrogen has an atomic weight (number of neutrons plus protons) of 14. Hydrogen has an atomic weight of 1. The number of atoms in a cubic meter is the same for both gases, so the factor of 14 simply reflects the larger nitrogen nucleus.

[25] Here is a quote from that article:

To four teenagers from the suburbs, Split Rock Falls was a magical place — cool water rushing between the granite walls of a mountain ravine, forming pools for hours of lazy summertime swimming.

On Tuesday afternoon the four men — Adam Cohen, 19; Jonah Richman, 18; Jordan Satin, 19; and David Altschuler, 18 — returned to their favorite childhood summer haunt to find it engorged by a summer of heavy rain. By the end of the day, all four men, each an experienced swimmer, was dead, drowned in the waters they knew well.

In what officials here described as one of the worst drowning accidents ever in the Adirondack State Park, all four died after Mr. Altschuler slipped off a narrow granite ledge into a foaming pool of water whipped into a frenzy by a tumbling waterfall. In a final act of friendship, Mr. Richman, Mr. Cohen and Mr. Satin, who had grown up

Submarines can adjust their "altitude" below the ocean surface by taking in and expelling water into their ballast tanks. When they take in water, the air in the tanks is replaced by the heavier water, and the average density of the submarine increases. This makes the submarine sink. The only thing that will stop the sinking is expulsion of water from the tanks. If the submarine sinks too far, then it is crushed by the weight of the water above it; that makes it even more dense, and so it sinks faster. That is called the hull crush depth. In the movie "Crimson Tide," that depth was 1800 feet, about 1/3 mile down. The submarine in that movie (the fictional USS Alabama) was able to save itself by getting its engine running, and using its short "wings" to get lift in way similar to the way that an airplane does, by moving forward and pushing water downward. A submarine can also push compressed air into its ballast tanks, driving out the water, and decreasing its average density.

Sperm whales are said to be able to dive as deep at 2 miles. Diving deep is easy, since as the whale goes deeper, any air in its lungs (remember, a whale is a mammal) is compressed, and that makes the whale less buoyant. So once the whale is denser than water, it will sink. Coming up is the hard part. Whales save enough energy in their effortless dives, to be able to swim back up to the surface.

## Air pressure on mountains, outside airplanes and satellites

Air pressure is simply the weight of the air above you. In any fluid or gas, the pressure is evenly distributed, so that the air at sea level will push an equal amount up, down, and sideways. As you go higher, there is less air above you, so the pressure decreases. At an altitude of 23,000 feet (4.3 miles, 6.9 km) the pressure is half of what it is at sea level. Go up another 23,000 feet – to 46,000 feet, and the pressure is reduced by another factor of 2 – to one quarter the pressure at sea level. That's a typical altitude for jet airplanes. And that rule continues; for every additional 23000 feet, the pressure (and the density of the air[26]) drops by another factor of two. You can not live at such a low density of air, and that is why airplanes are pressurized. You can live at that pressure if the "air" you are given is pure oxygen (rather than 20%), and that is what you would get from the emergency face masks that drop down in airline seats if the cabin ever loses pressure.

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together on Long Island, jumped after him to try to save his life, police and officials said. The laws of physics were against them, though.

"They call it a drowning machine," said Lt. Fred J. Larow, a forest ranger with the State Department of Environmental Conservation, who helped recover the bodies here, about 20 miles east of Lake Placid. "The water was so turbulent and aerated that there was no way they could stay above water. Even the strongest swimmer in the world couldn't have survived it."

[26] The air density doesn't drop quite that much because the air up there is cooler.

The equation for this can be written in a simple way. Let P be the relative pressure at an altitude H, compared to the pressure at sea level. So for H = 0, P = 1. Then the equation is:

$$P = (1/2)^{H/23000} \text{ for H in feet}$$
$$P = (1/2)^{H/6.9} \text{ for H in km}$$

To work this out, take the altitude (say in km), divide it by 6.9 feet, and write down your answer. Then multipy 1/2 by itself that many times. That can easily be done on a calculator. For example, consider a “low earth orbit” satellite, at 200 km above sea level. A/6.9 = 200/6.9 = 28.99 ≈ 29. Now we have to multiply (1/2) by itself 29 times. That gives

$$P = (1/2)^{29} = 2E\text{-}9 = 0.000000002$$

That’s a pressure of two billionths times as small as at sea level. Satellites need this low pressure to avoid being slowed down by collisions with air.

Because the density of air decreases with altitude, a helium balloon will not rise forever. Eventually it reaches an altitude at which the outside air has the same density as the helium (with the weight of the gondola averaged in), and then it stops rising. I noticed this as a child, when I was disappointed to see that the helium-filled balloon I had released did not go all the way to space, but went high up and then stopped rising.

That’s why balloons are not a possible way to get to space.

## Convection: thunderstorms

When air is heated near the ground, its density is reduced, and it tends to rise, just like a hot air balloon. unconstrained by any balloon, the air expands as it rises, and an interesting result is that it will remain less dense than the surrounding air until it reaches the “tropopause”, the level in which ozone absorbs sunlight causing the surrounding air to be warmer. When it reaches the tropospause, its density is no longer less than that of the surrounding air, so the air stops rising.

On a summer day, when thunderstorms are growing, it is easy to spot the tropopause. It is the altitude at which the thunderstorms stop rising, and begin to spread out laterally. The tropopause is a very important layer in the atmosphere. It is the location of the “ozone layer” which protects us from cancer-producing ultraviolet light. We’ll talk more about his layer in Chapter 8 Waves (because of the important effect it has on sound), and in Chapter 10 Invisible light, when we discuss ultraviolet radiation and its effects.

## Optional section:
## Angular momentum and torque

In addition to ordinary momentum (mass times velocity), there is another kind of momentum that physicists and engineers find enormously useful in their calculations called angular momentum. Angular momentum is similar to ordinary momentum but it most valuable for motion that is circular, i.e. rotation. If an object of mass $M$ is spinning in a circle of radius $R$, moving at velocity $v$, then its angular momentum $L$ is:

$$L = M\, v^2\, R$$

What makes angular momtum so useful is that, like ordinary momentum, when there are no external forces on an object, it is "conserved", i.e. its value doesn't change. Have you ever spun on iceskates? Actually, ice skates are not necessary – just stand in a spot and start spinning with your arms stretched out. (If you've never done this, then I strongly recommend you try it right now. Then rapidly pull your arms in. (You'll never forget the experience, and you can have great fun intertaining children with it. ) To the surprise of most people, they will suddenly spin much faster. You can predict that from the angular momentum equation. If the angular momentum L is the same before and after the arms are pulled in, and the mass of the arms $M$ is the same, then $v^2\, R$ must be the same. If $R$ gets smaller, then $v$ must get bigger.

Angular momentum conservations also explains why water leaving a tub through a narrow drain begins to spin. In fact, it is very unlikely that the water in the tub wasn't already spinning, at least a little bit. But when the distance to the drain (R, in the equation) gets small, the v in the equation gets very big. Incidentally, it is not true that drains or toilets drain differently in the northern and southern hemispheres. A similar effect occurs in hurricanes and tornadoes. Air being sucked into the center (where there is a low pressure due to air moving upward) spins faster and faster, and that's what gives the high velocities of the air in these storms. The air in hurricanes gets its intial spin from the spin of the Earth, and then they amplify it by the angular momentum effect. So hurricanes really do spin in different directions in the Southern and Northern Hemispheres.

Conservation of angular momentum can also be used to understand how a cat, dropped from an upside down position, can still land on his feet. (Don't try this one at home! I never have, but I've seen a movie...) If he spins his legs in a circle, his body will move in the opposite direction, keeping his total angular momentum equal to zero. Astronauts do this trick when they want to reorient themselves in space. Spin an arm in a circle, and your body will move in the opposite direction. You can try that trick on ice skates too.

The conservation of angular momentum has other useful applications. It helps keep a bicycle wheel from falling over, when the wheel is spinning. It can also cause a difficulty: if kinetic energy is stored in a spinning wheel (usually called a "flywheel"), then the angular momentum makes it difficult to change the direction that the wheel is spinning. This makes its use for energy storage in moving vehicles, such as buses, problematical.  It is often addressed by having two flywheels spinning in opposite diretions, so although energy is stored, the total angular momentum is zero.

Angular momentum can be changed by a suitable application of an outside force.  The required geometry is that the force must act obliqely, at a distance.  We define torque as the tangential component of the force (the oblique part) times the distance to the center. So, for example, to start a bicycle wheel spinning, you can't just push on the rim in a radial manner.  You have to push tangentially.  That's called torque.  The law relating torque and angular momentum is this: the rate of change of angular momentum is numerically equal to the torque.

You can probably see why mastery of the equations of angular momentum is very useful to engineers and physicists in simplifying their calculations.

END OF CHAPTER

## Quick review

All things that have mass attract each other with a force given by the gravity equation:

$$F = G\ M_1\ M_2/r^2$$

This force makes apples fall from trees, and keeps the moon in its orbit.

A force on an object makes it accelerate by an amount (given in m/s per sec) of

$$F = M\ a$$

But objects accelerate only if the total force is not zero.  When an object moves through air, the air resistance tends to slow it down.  For a falling object, it has a terminal velocity that is typically 15 mph (for a parachutte) to 100 mph (for a person).  The force of air is proportional to the square of the velocity, so it becomes very large at high velocities. Overcoming this force is a major problem for automobiles, cars, and satellites.

The energy it takes to push an object with a force F (measured in kg) over a distance D is

$$E = g\ F\ D$$

where $g = 9.8 \approx 10$ is a constant known as the "acceleration of gravity".
When you travel in a circular path, there is an apparent force pushing you away from the center that we call the centrifugal force:

$$F = M\ v^2/R$$

Satellites travel in circular orbits when the centrifugal force exactly balances the attraction of gravity.  An astronaut feels "weighttless" because of the balance of forces. Low earth satellites take about 90 minutes to go around; they can not be stationary. Geostationary satellites take 1 day to go around, so they stay above the same location on Earth.  The moon takes a month to go around.

To get into orbit requires a velocity of about 5 miles per second.  To escape completely takes 7 miles per second, known as the "escape velocity." Black holes occur when the escape velocity exceeds the speed of light.

Gravity measurements have practical applications.  Since oil is lighter than rock, it has a weaker gravity, and that fact has been used to locate it.

When a gun is fired, the bullet goes forward and the gun goes backward. This is an example of the conservation of momentum. Other examples: Rockets go forward (very inefficiently) by shooting burnt fuel backwards. Airplanes fly by pushing air dowwards. To reach high velocities efficiently, we may some day use electric methods. Objects float when their density is less than that of the fluid or gas they are in. That includes boats and balloons. Hot air rises because it is less dense than the surrounding air, and that happens for hot air balloons and thunderstorms. The density and pressure of air decreases with altitude according to a halving rule:

$$P = (1/2)^{H/6.9}$$

where the height H is expressed in km.

Angular momentum (a momentum that applies to circular motion) is also conserved, and that causes contracting objects to speed up. Examples include sink drains, hurricanes, and tornadoes.

## Numerical problems

If you are standing on the Equator, you are spinning around in a circular path. If you are standing on the North Pole, you are not. So just from the centrifugal force at the Equator, you should weigh less. Estimate how much less you will weigh. Do you think this effect can be measured? Would it be a useful way to measure your lattitude?

How do we know the mass of the sun? We can get it by knowing two facts: the sun is 150 million km = 1.5E11 m from the Earth, and it orbits in 1 year. By setting the force of gravity equal to the centrifugal force, show that the mass of the Earth is 2E30 kg.

Estimate the velocity of recoil of a truck, when it is hit by a mosquito. Assume (from the point of view of the truck), the mosquito (weighing 2.6 milligrams) is moving at 60 mph = 27 meters per second. Assume the truck weighs 5 metric tons = 5000 kg. (Hint: use the same equation we used for the comet hitting the Earth, except let the 2.6 milligram mosquito represent the comet.)

Which is greater, the gravitational attraction between two peole who are talking to each other, or the gravitational attraction between two ants that are standing right next to each other?

## Qualitative questions

At high velocities, most of the fuel used in an automobile is used to overcome

( ) gravity
( ) momentum
( ) air resistance
( ) buoyancy

Airplanes fly by

( ) pushing fuel downward
( ) pushing fuel backwards

( ) pushing fuel upwards
( ) pushing air downwards

At at altitude of 200 km, the downward force of gravity on an Earth satellite is
( ) the same as on the surface of the Earth
( ) a little bit weaker
( ) zero
( ) a little bit stronger

In the southern hemisphere, sinks drain
( ) usually clockwise
( ) the same way they do in the Northern Hemisphere
( ) the answer is different in the Eastern and Western Hemisopheres.

A rail gun
( ) accelerates rails
( ) can shoot bullets faster than ordinary guns
( ) doesn't really accelerate things
( ) uses ions for propulsion

The Sun would be a black hole if it were squeezed into a radius of
( ) 2 miles
( ) 2,000 miles
( ) 2 million miles
( ) It already is a black hole

The method that would take the least energy to get something to space is:
( ) lift it by elevator (if the elevator existed)
( ) launch it in a one-stage rocket
( ) fly it in a balloon
( ) launch it in a three-stage rocket

If an artificial satellite orbited the Earth at the distance of 240,000 miles (the preseent distance to the moon), it would orbit the Earth in a period of
( ) 90 minutes
( ) 1 day
( ) 1 week
( ) 1 month

The time it takes a geosynchronous satellite to orbit the Earth is:
( ) 90 minutes
( ) 1 day
( ) 1 week
( ) 1 month

A typical terminal velocity for a high diver is closest to:

( ) 1 mph
( ) 10 mph
( ) 100 mph
( ) 1000 mph

The force of gravity between two people standing next to each other is
( ) zero
( ) too small to measure
( ) small but measureable
( ) greater than 1 kg

For a low Earth satellite (altitude 200 km), the force of gravity is
( ) zero
( ) a few percent less than on the Earth's surface
( ) exactly what it is on the Earth's surface
( ) about half of what it is on the Earth's surface

The altitude of a geosynchronous satellite is closest to:
( ) 200 miles
( ) 26,000 miles
( ) 2,000,000 miles
( ) 93,000,000 miles

Rockets fly by
( ) using antigravity
( ) pushing air downwards
( ) pushing fuel downwards
( ) being lighter than air

Ballons rise until
( ) the surrounding air becomes too dense
( ) the surrounding air beomes too thin ( not dense)
( ) the sourounding air becomes too cold
( ) they reach space

You can see where the tropopause is by
( ) looking at Earth satellites
( ) seeing how high birds fly
( ) looking at the tops of thunderstorms
( ) seeing where lightning comes from

Ice skaters spin faster by pulling in their arms.  This illustrates that
( ) momentum is conserved
( ) angular momentum is conserved
( ) energy is conserved
( ) angular energy is conserved

Someone firing a rifle is pushed back by the rifle. This illusrates that

( ) momentum is conserved
( ) angular momentum is conserved
( ) energy is conserved
( ) angular energy is conserved

(more need to be written)

## Essay questions

Automoblies and trucks are sometimes designed with a tapered ("aerodynamic") rather than a blunt front surface. Why is this? What does the tapered front accomplish? How important is it?

Artificial Earth satellites fly at different altitudes, depending on what they need to do. Decribe the differences between low Earth satellites and geosynchronous satellites. For which applications would you use each?

(more will be written)